Date: 9-2-2021

Trigonometric · Imp Formulas:-

• Radian and Degree Measures of Angles

$1 \text{rad} = \frac{180°}{\pi}$ & $1° = \frac{\pi}{180}$

Angle degree	0	30	45	60	90	180	270	360
Angle radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

• Signs of Trigonometric Function:

Quadrant	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$	$\sec\alpha$	$\mathrm{cosec}\,\alpha$
I	+	+	+	+	+	+
II	+					+
III			+	+		
IV		+			+	

• Trigonometric Functions of common angles:

Angles	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$	$\sec\alpha$	$\mathrm{cosec}\,\alpha$
0	0	1	0	∞	1	∞
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
45	$1/\sqrt{2}$	$1/\sqrt{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$
90	1	0	∞	0	∞	1
120	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	-2	$\frac{2}{\sqrt{3}}$
180	0	-1	0	∞	-1	∞
270	-1	0	∞	0	∞	-1
360	0	1	0	∞	1	∞

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Most Imp Formulas:-

i- $\sin^2\theta + \cos^2\theta = 1$, iv- $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

ii- $\sec^2\theta - \tan^2\theta = 1$, v- $\cot\theta = \frac{\cos\theta}{\sin\theta}$

iii- $\csc^2\theta - \cot^2\theta = 1$, vi- $\tan\theta = \frac{1}{\cot\theta}$

vii- $\cos\theta = \frac{1}{\sec\theta}$, viii- $\csc\theta = \frac{1}{\sin\theta}$

• Addition and Subtraction Formulas:

i- $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

ii- $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

iii- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

iv- $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

v- $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$, vi- $\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

vii- $\cot(\alpha+\beta) = \frac{\cot-\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$, viii- $\cot(\alpha-\beta) = \frac{1+\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$

• Half angle Formulas:

i- $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$, ii- $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

iii- $\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$, iv- $\cot\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$

• Double angle Formulas:

i- $\sin 2\alpha = 2\sin\alpha\cos\alpha$

ii- $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$

iii- $\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2}{\cot\alpha - \tan\alpha}$

iv- $\cot 2\alpha = \frac{\cot^2\alpha - 1}{2\cot\alpha} = \frac{\cot\alpha - \tan\alpha}{2}$

• Half angle Tangent Identities:-

i- $\sin\alpha = \frac{2\tan\alpha/2}{1+\tan^2\frac{\alpha}{2}}$, ii- $\cos\alpha = \frac{1-\tan^2\alpha/2}{1+\tan^2\alpha/2}$

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iii- $\tan\alpha = \frac{2\tan\alpha/2}{1-\tan^2\alpha/2}$, iv- $\cot\alpha = \frac{1-\tan^2\alpha/2}{2\tan\alpha/2}$

• Transforming of Trigonometric Expressions to Product

i. $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$

ii. $\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

iii. $\cos\alpha + \cos\beta = 2\cos\alpha+\beta/2 \cos\alpha-\beta/2$

iv. $\cos\alpha - \cos\beta = -2\sin\alpha+\beta/2 \sin\alpha-\beta/2$

v. $\tan\alpha + \tan\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}$, vi- $\tan\alpha - \tan\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}$

vii. $\cot\alpha + \cot\beta = \frac{\sin(\alpha+\beta)}{\sin\alpha\cdot\sin\beta}$, viii- $\cot\alpha - \cot\beta = \frac{\sin(\alpha-\beta)}{\sin\alpha\cdot\sin\beta}$

ix. $\cos\alpha + \sin\alpha = \sqrt{2}\cos\left(\frac{\pi}{4}-\alpha\right) = \sqrt{2}\sin\left(\frac{\pi}{4}+\alpha\right)$

x. $\cos\alpha - \sin\alpha = \sqrt{2}\sin\left(\frac{\pi}{4}-\alpha\right) = \sqrt{2}\cos\left(\frac{\pi}{4}+\alpha\right)$

xi. $\tan\alpha + \cot\beta = \frac{\cos(\alpha-\beta)}{\cos\alpha\cdot\sin\beta}$, xii- $\tan\alpha - \cot\beta = \frac{\cos(\alpha+\beta)}{\cos\alpha\cdot\sin\beta}$

xiii. $1+\cos\alpha = 2\cos^2\frac{\alpha}{2}$, xiv- $1-\cos\alpha = 2\sin^2\frac{\alpha}{2}$

xv- $1+\sin\alpha = 2\cos^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$, xvi- $1-\sin\alpha = 2\sin^2\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$

• Transforming of Trigonometric Expressions to sum.

i- $2\sin\alpha\cdot\sin\beta = \cos(\alpha+\beta) - \cos(\alpha-\beta)$

ii. $2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

iii. $2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$

iv. $2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$

v. $\tan\alpha\tan\beta = \frac{\tan\alpha+\tan\beta}{\cot\alpha+\cot\beta}$, vi. $\cot\alpha\cdot\cot\beta = \frac{\cot\alpha+\cot\beta}{\tan\alpha+\tan\beta}$

vii. $\tan\alpha\cot\beta = \frac{\tan\alpha+\cot\beta}{\cot\alpha+\tan\beta}$

Date:

Powers of Trigonometric Functions:-

i. $\sin^2\theta = \frac{1-\cos 2\theta}{2}$, ii $\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$

iii. $\sin^5\theta = 10\sin\theta - 5\sin 3\theta + \sin 5\theta/16$

iv $\sin^6\theta = 10 - 15\cos 2\theta + 6\cos 4\theta - \cos 6\theta/32$

v. $\cos^2\theta = 1 + \cos 2\theta/2$, vi. $\cos^3\theta = 3\cos\theta + \cos 3\theta/4$

vii. $\cos^4\theta = \cos 4\theta + 4\cos 2\theta + 3/8$

viii. $\sin^4\theta = \cos 4\theta - 4\cos 2\theta + 3/8$

ix $\cos^5\theta = 10\cos\theta + 5\sin 3\theta + \cos 5\theta/16$

x. $\cos^6\theta = 10 + 15\cos 2\theta + 6\cos 4\theta + \cos 6\theta/32$

Sine Rule:-

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule:-

i. $a^2 = c^2 + b^2 - 2cb\cos\alpha$ ii. $b^2 = a^2 + c^2 - 2ac\cos\beta$

iii. $c^2 = a^2 + b^2 - 2ab\cos\gamma$

Negative angle identities:-

i. $\sec(-\theta) = +\sec\theta$, ii. $\cos(-\theta) = +\cos\theta$

ii. $\mathrm{cosec}(-\theta) = -\mathrm{cosec}\,\theta$, iv. $\sin(-\theta) = -\sin\theta$

iv. $\cot(-\theta) = -\cot\theta$, vi. $\tan(-\theta) = -\tan\theta$

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"Derivatives:"

- Power rule:- $\frac{d}{dx}[x^n] = nx^{n-1}$
- Chain rule:- (i) $\frac{d}{dx}[f(g(x))] = f'(g(x)).g'(x)$

  (ii) $\frac{dy}{dx} = \frac{dy}{dm} \times \frac{dm}{dx}$
- Product rule:-

$$[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

- Quotient rule:-

$$[f(x)/g(x)] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

- Trigonometric functions:-

i- $\frac{d}{dx}(\sin x) = \cos x$, iv. $\frac{d}{dx}(\operatorname{cosec} x) = -\cot x.\operatorname{cosec} x$

ii- $\frac{d}{dx}(\cos x) = -\sin x$, v. $\frac{d}{dx}(\sec x) = \tan x.\sec x$

iii- $\frac{d}{dx}(\tan x) = \sec^2 x$, vi. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

- Inverse Trigonometric functions:-

i- $\frac{d}{dx}(\sin^{-1}x) = 1/\sqrt{1-x^2}$, ii. $\frac{d}{dx}(\cos^{-1}x) = -1/\sqrt{1-x^2}$

iii- $\frac{d}{dx}(\tan^{-1}x) = 1/1+x^2$, iv- $\frac{d}{dx}(\cot^{-1}x) = -1/1+x^2$

v- $\frac{d}{dx}(\sec^{-1}x) = 1/|x|\sqrt{x^2-1}$, vi- $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = -1/|x|\sqrt{x^2-1}$

- Hyperbolic functions:-

i- $\frac{d}{dx}(\sinh x) = \cosh x$, ii- $\frac{d}{dx}(\cosh x) = -\sinh x$

iii- $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$, iv- $\frac{d}{dx}(\operatorname{sech} x) = +\operatorname{sech} x.\tanh x$

v- $\frac{d}{dx}(\coth x) = -\operatorname{cosec}^2 h^2 x$, vi- $\frac{d}{dx}(\operatorname{cosech} x) = -\operatorname{cosech} x.\coth x$

i- $\sinh x = e^x - e^{-x}/2$, ii $\cosh x = e^x + e^{-x}/2$

iii. $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, iv $\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

v. $\operatorname{cosech} x = \frac{2}{e^x - e^{-x}}$, vi $\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$

- Derivatives of inverse hyperbolic functions:-

i- $\frac{d}{dx}(\sinh^{-1}x) = 1/\sqrt{1+x^2}$, ii- $\frac{d}{dx}(\cosh^{-1}x) = 1/\sqrt{x^2-1}$

iii. $\frac{d}{dx}(\tanh^{-1}x) = 1/1-x^2, |x|<1$, iv. $\frac{d}{dx}(\coth^{-1}x) = 1/1-x^2, |x|>1$

v. $\frac{d}{dx}(\operatorname{cosech}^{-1}x) = -1/|x|\sqrt{1+x^2}$, vi- $\frac{d}{dx}(\operatorname{sech}^{-1}x) = -1/x\sqrt{1-x^2}$

i. $\sinh^{-1}x = \ln(x+\sqrt{x^2+1})$, ii $\cosh^{-1}x = \ln(x+\sqrt{x^2-1})$

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- General Derivative Form:-

i. $\frac{d}{dx}(C) = 0$, ii. $\frac{d}{dx}(x) = 1$, iii. $\frac{d}{dx}(x^n) = nx^{n-1}$

iv. $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$, v. $\frac{d}{dx}(\ln x) = \frac{1}{x}$, vi. $\frac{d}{dx}(e^x) = e^x$

- Binomial Theorm:-

$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \dots \binom{n}{n}a^0 b^n$

- Binomial series:-

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$

- Maclaurin series:-

$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{iv}(0)}{4!}x^4 + \dots + \frac{f^{(n)}(0)}{n!}x^n + \dots$

- Tailor series:-

$f(x+h) = f(x) + \frac{f'(x)h}{1!} + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \dots + \frac{f^{(n)}(x)}{n!}h^n + \dots$

- $\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$, $\coth^{-1}x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$

$\operatorname{sech}^{-1}x = \ln\left(\frac{1}{x} + \frac{\sqrt{1-x^2}}{x}\right)$, $\operatorname{cosech}^{-1}x = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right)$

- $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$
- $\lim_{x \to 0} \frac{\sqrt{x+a} - \sqrt{a}}{x} = \frac{1}{2\sqrt{a}}$
- $\lim_{x \to \infty} \left(1 + \frac{1}{n}\right)^n = e$
- $\lim_{x \to 0} (1+x)^{1/n} = e$
- $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$
- $\lim_{x \to 0} \left(\frac{e^x - 1}{x}\right) = \log_e e = 1$

- Perimeter of square $= 4x$, Area of circle $= \pi r^2$
- Area of square $= x^2$
- Circumference of circle $= 2\pi r$
- Volume of cube $= x^3$
- Sandwitch Theorem:-

$\lim_{x \to c} f(x) = L$ & $\lim_{x \to c} h(x) = L$ & $\lim_{x \to c} g(x) = L$

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CH-3: Integration

DATE: 27-5-2021

. General integration form:-

$\Rightarrow \int 0\,dx = C$

$\Rightarrow \int 1\,dx = x$

$\Rightarrow \int x^n dx = \frac{x^{n+1}}{n+1} + C$

$\Rightarrow \int \frac{1}{x} dx = \ln|x| + C$

$\Rightarrow \int a^x dx = \frac{a^x}{\ln a} + C$

$\Rightarrow \int e^x dx = e^x + C$

$\Rightarrow \int e^{2x} dx = \frac{e}{2} + C$

1:- $\int \sin x\,dx = -\cos x + C$

2: $\int \cos x\,dx = \sin x + C$

3: $\int \sec^2 x\,dx = \tan x + C$

4: $\int \operatorname{cosec}^2 x\,dx = -\cot x + C$

5: $\int \sec x \tan x\,dx = \sec x + C$

6: $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$

7: $\int \sin nx\,dx = \frac{-\cos nx}{n} + C$

8: $\int e^{nx} dx = \frac{e^{nx}}{n} + C$

9: $\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

10: $\int \frac{-1}{1+x^2} dx = \cot^{-1} x + C$

11: $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$

12: $\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C$

13: $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

14: $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \frac{x}{a} + C$

15: $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a} \sec^{-1} \frac{|x|}{a} + C$

16: $\int \tan x\,dx = -\ln|\cos x| + C$

17: $\int \cot x\,dx = \ln|\sin x| + C$

18: $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

19: $\int \operatorname{cosec} x\,dx = \ln|\operatorname{cosec} x - \cot x| + C$

20: $\sqrt{a^2-x^2} \Rightarrow x = a\sin\theta$

21: $\sqrt{x^2-a^2} \Rightarrow x = a\sec\theta$

22: $\sqrt{a^2+x^2} \Rightarrow x = a\tan\theta$

23: $\int \frac{1}{\sqrt{a^2+x^2}} dx = \ln|x + \sqrt{a^2+x^2}| + C$

24: $\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln|x + \sqrt{x^2-a^2}| + C$

Date: 10-4-2021

CH-4 "Analytic Geometry"

Quadrant I: $x>0, y>0$

Quadrant II: $x<0, y>0$

Quadrant III: $x<0, y<0$

Quadrant IV: $x>0, y<0$

Phythagorean Theorem:

$$|AB|^2 + |BC|^2 = |CA|^2$$

Distance Formula:

$$d = |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Section Formula:

i) When the ratio $k_1 : k_2$ is internal.

$$\left(\frac{k_1 x_2 + k_2 x_1}{k_1 + k_2}, \frac{k_1 y_2 + k_2 y_1}{k_1 + k_2}\right)$$

ii) When the ratio $k_1 : k_2$ is external:

$$\left(\frac{k_1 x_2 - k_2 x_1}{k_1 - k_2}, \frac{k_1 y_2 - k_2 y_1}{k_1 - k_2}\right)$$

Midpoint Formula:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Centroid Formula:

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

Equation of Translation:

$$X = x - h \quad , \quad Y = y - k$$

Equation of Rotation:

$$X = x\cos\theta + y\sin\theta$$

$$Y = y\cos\theta - x\sin\theta$$

Slope $= m = \tan\alpha$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ (if two points are given)

Date: 10-4-2021

- If l is horizontal, m is zero
- If l is vertical, m is undefined
- If $0 < \alpha < 90°$, m is positive
- If $90° < \alpha < 180°$, m is negative

$m = -\frac{a}{b}$ if line $(ax + by + c \neq 0)$ is given

Two lines are parallel if $m_1 = m_2$ also $a_1 b_2 - a_2 b_1 = 0$

Two lines are perpendicular $m_1 \cdot m_2 = -1$

Slope of AB = Slope of BC

Thus A, B and C are Collinear

Slope intercept form $y = mx + c$

Two intercept form $\frac{x}{a} + \frac{y}{b} = 1$

Equation of form Line:

$$(y - y_1) = m(x - x_1)$$

Symmetric from $\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha} = r$

Normal from $x\cos\alpha + y\sin\alpha = P$

Distance = d (from one point to lie) $= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Area of triangle $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$, $m_1 + m_2 = \frac{-2h}{b}$ & $m_1 m_2 = \frac{a}{b}$

$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$

$h^2 - ab = 0$ then lines are concident

$a + b = 0$ then $\theta = 90°$

Joint equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Homogeneous equation $ax^2 + 2hxy + by^2 = 0$

Scanned with CamScanner

CH 6: Conic Sections

DATE:

Equation of circle in standard form:

$$(x-h)^2+(y-k)^2=r^2 \quad \because C(h,k), \text{radius}-r$$

Equation of circle when centre is at origin:

$$x^2+y^2=r^2$$

Parametric equations:

$x = r\cos\theta$, $y = r\sin\theta$

Equation of circle in General form:

$$x^2+y^2+2gx+2fy+c=0$$

Centre $(-g,-f)$, Radius $r = \sqrt{g^2+f^2+c}$

1. A circle passing through three non-collinear points
2. A circle passing through two points having its centre on a given line.

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

3. A circle passing through two points and equation of tangent at one of these point known
4. A circle passing through two points and touching a given line.

Equation of Tangent:

$$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$$

Equation of normal:

$$(y-y_1)(x_1+g)=(x-x_1)(y_1+f)$$

Tangent to circle:

$$c^2=a^2(1+m^2) \quad \text{or} \quad c=\pm a\sqrt{(1+m^2)}$$

Length of tangent:

$$\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$$

یونٹ نمبر 7 : ویکٹرز DATE: 18/6/2021

=> Vector = $\vec{AB}$ or $\underline{U}$ => Scalar = AB or U

=> Magnitude = $|\vec{AB}|$ => Unit vector => $\hat{\underline{v}} = \frac{\underline{v}}{|\underline{v}|}$

=> Triangular Law of addition $\vec{AB} + \vec{BC} = \vec{AC}$

=> Equal vectors $|\vec{AB}| = |\vec{CD}|$

=> Position vector = $\vec{OP}$

=> Ratio Formula $\underline{r} = \frac{q\underline{a} + p\underline{b}}{p+q}$

=> Direction angles α, β, γ

=> Direction cosines $\cos\alpha = \frac{x}{r}$, $\cos\beta = \frac{y}{r}$, $\cos\gamma = \frac{z}{r}$

=> Scalar or Dot product of $\underline{u}$ and $\underline{v}$

$$\underline{u} \cdot \underline{v} = |\underline{u}||\underline{v}| \cos\theta$$

=> Vector or cross product of $\underline{u}$ and $\underline{v}$

$$\underline{u} \times \underline{v} = |\underline{u}||\underline{v}| \sin\theta \, \hat{n}$$

=> Angle between two vectors:

$$\Rightarrow \cos\theta = \frac{\underline{u} \cdot \underline{v}}{|\underline{u}||\underline{v}|}$$

=> Angle b/w two vectors:

$$\Rightarrow \sin\theta = \frac{|\underline{u} \times \underline{v}|}{|\underline{u}||\underline{v}|}$$

=> Perpendicular $\underline{u} \cdot \underline{v} = 0$

=> Parallel $\underline{u} \times \underline{v} = 0$

$$\Rightarrow \underline{u} \times \underline{v} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}, \text{ where } \begin{array}{l} \underline{u} = u_1\underline{i} + u_2\underline{j} + u_3\underline{k} \\ \underline{v} = v_1\underline{i} + v_2\underline{j} + v_3\underline{k} \end{array}$$

=> Area of parallelogram ABCD = $|\vec{AB} \times \vec{AC}|$

=> Area of Triangle ABC = $\frac{1}{2}|\vec{AB} \times \vec{AC}|$

=> Volume of parallelepiped = $\underline{u} \cdot (\underline{v} \times \underline{w})$

=> $\underline{u}, \underline{v}, \underline{w}$ are coplanar if $\underline{u} \cdot (\underline{v} \times \underline{w}) = 0$

Imperial Pro

=> Volume of Tetrahedron $= \frac{1}{6}(\underline{u}\cdot(\underline{v}\times\underline{w}))$

=> Work done $= \underline{F}\cdot\underline{D}$

=> Moment of force $= \underline{r}\times\underline{F}$

=> $\underline{u}\cdot\underline{v}\times\underline{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

=> $\underline{u}\cdot\underline{v}\times\underline{w} = \underline{v}\cdot\underline{w}\times\underline{u} = \underline{w}\cdot\underline{u}\times\underline{v}$

Hammad

23/07/21